Radiative decays of scalar mesons $f_0(980)$ and $a_0(980)$ into $\rho(\omega)\gamma$ in the local Nambu-Jona-Lasinio model

M. K. Volkov,* Yu. M. Bystritskiy,† and E. A. Kuraev‡
Joint Institute for Nuclear Research, Dubna, Russia
(Dated:)

In the framework of the local Nambu-Jona-Lasinio model the radiative decay widths of the scalar mesons $f_0(980)$ and $a_0(980)$ into $\rho\gamma$ and $\omega\gamma$ are calculated. The contributions of the quark loops and the meson loops are taken into account. For the radiative decays of the scalar meson $f_0(980)$ the contribution of the meson loops plays the dominant role. On the other side for the radiative decays of the scalar meson $a_0(980)$ the main contribution is given by the quark loops.

I. INTRODUCTION

Recently two works were published devoted to the description of the vector meson decay $\phi \to f_0\gamma$ [1] and to two photon decays of scalar mesons $\sigma(600) \to \gamma\gamma$ and $f_0(980) \to \gamma\gamma$ [2]. These decays were described in the framework of the local Nambu-Jona-Lasinio (NJL) model where both – the quark loop and the meson loop – were taken into account. The satisfactory agreement with the experimental data was obtained.

The comparison with the other models was performed. First of them is the model which considers the scalar mesons as a four-quark states [3]. In other model the scalar mesons are treated as a kaon molecule [4]. It was shown that using the NJL model in $1/N_c$ approximation (where N_c is the number of colors) we gain the qualitative agreement with the predictions of these phenomenological models for decays of f_0-mesons.

Here we would like to investigate the radiative decays of scalar mesons $f_0(980)$ and $a_0(980)$ into light vector mesons: $f_0(980) \to \rho(\omega)\gamma$, $a_0(980) \to \rho(\omega)\gamma$.

Unfortunately now we do not have any experimental data for this decays. However satisfactory description of $\gamma\gamma$ channels and $\phi \to f_0(980)\gamma$ decay in the framework of NJL model allows us to hope that we could obtain a reasonable predictions for the decays mentioned above.

In future we are going to use our results for descriptions of processes $e\bar{e} \to SV$, where $V = \rho, \omega$ and $S = f_0, a_0$.

*Electronic address: volkov@theor.jinr.ru
†Electronic address: bystr@theor.jinr.ru
‡Electronic address: kuraev@theor.jinr.ru

II. LAGRANGIAN OF THE NJL MODEL

The lagrangian of interaction of mesons and quarks whithin the NJL model has the form [5, 6]:

$$\mathcal{L}_{int} = \bar{q}\left[eQ\hat{A} + g_u\lambda_u\sigma_u + g_s\lambda_s\sigma_s + g_u\lambda_3 a_0 + i\gamma_5 g_\pi\left(\lambda_{\pi^+}\pi^+ + \lambda_{\pi^-}\pi^-\right) + \right.$$
$$\left. + i\gamma_5 g_K\left(\lambda_{K^+}K^+ + \lambda_{K^-}K^-\right) + \frac{g_\rho}{2}\left(\lambda_3\hat{\rho}_0 + \lambda_u\hat{\omega}\right)\right]q, \qquad (1)$$

where $\bar{q} = \left(\bar{u}, \bar{d}, \bar{s}\right)$, and u, d, s is the quark fields, $Q = \mathrm{diag}\,(2/3, -1/3, -1/3)$ is the quark electric charges matrix, e is the elementary electric charge ($e^2/4\pi = \alpha = 1/137$), $\lambda_u = \left(\sqrt{2}\lambda_0 + \lambda_8\right)/\sqrt{3}$, $\lambda_s = \left(-\lambda_0 + \sqrt{2}\lambda_8\right)/\sqrt{3}$, where λ_i is the well-known Gell-Mann matrixes and $\lambda_0 = \sqrt{2/3}\,\mathrm{diag}\,(1, 1, 1)$. That is

$$\lambda_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_s = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\sqrt{2} \end{pmatrix}, \qquad \lambda_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_{\pi^+} = \begin{pmatrix} 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad \lambda_{\pi^-} = \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$
$$\lambda_{K^+} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \sqrt{2} & 0 & 0 \end{pmatrix}, \qquad \lambda_{K^-} = \begin{pmatrix} 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Vertex constants from the lagrangian (1) are defined in a following way:

$$g_{\sigma_u} = \left(4I^\Lambda(m_u, m_u)\right)^{-1/2},\ g_{\sigma_s} = \left(4I^\Lambda(m_s, m_s)\right)^{-1/2},$$
$$g_\pi = \sqrt{Z_\pi}g_{\sigma_u},\ g_K = \sqrt{Z_K}\left(4I^\Lambda(m_u, m_s)\right)^{-1/2}, \qquad (2)$$

where

$$I^\Lambda(m_1, m_2) = \frac{N_c}{(2\pi)^4}\int d^4k\frac{\theta(\Lambda^2 - k^2)}{(k^2 + m_1^2)(k^2 + m_2^2)}$$
$$= \frac{3}{(4\pi)^2}[m_2^2\ln\left(\frac{\Lambda^2}{m_2^2} + 1\right)$$
$$-m_1^2\ln\left(\frac{\Lambda^2}{m_1^2} + 1\right)]/(m_2^2 - m_1^2). \qquad (3)$$

These integrals are written in Euclidean space. The constituent quark masses $m_u = m_d = 263$ МэВ, $m_s = 407$ МэВ and the cut-off parameter $\Lambda = 1.27$ ГэВ are taken

from the paper [1]. As a result we get the following values for the coupling constants: $g_{\sigma_u} = 2.42$, $g_{\sigma_s} = 3.0$; where Z_π and Z_K are the factors which takes into account the transitions of pseudoscalar mesons into axial-vector ones: $Z_\pi = (1 - \frac{6m_u^2}{M_{a_1}^2})^{-1}$, $Z_K = (1 - \frac{3(m_u+m_s)^2}{2M_{K_1}^2})^{-1}$ [5], where $M_{a_1} = 1260$ МэВ, $M_{K_1} = 1403$ МэВ are the masses of axial-vector mesons [7]. In our calculation we assume that $Z_\pi \approx Z_K = Z = 1.4$. As a result we get $g_\pi \approx 2.9$, $g_K \approx 3.3$. $g_\rho = \sqrt{6} g_{\sigma_u} = 5.94$ is the constant of $\rho \to 2\pi$ decay [5]. σ_u and σ_s are the isoscalar scalar mesons in the case of ideal mixing, which suppose that σ_u consists of light quarks u and d only and σ_s consists of strange quarks s. Scalar mesons f_0 and σ are the mixtures of these two pure quark states with the mixing angle $\alpha = \theta_0 - \theta$, where $\theta_0 = 35.3°$ is the angle of the ideal singlet-octet mixing and $\theta = 24°$ is the angle of real mixing which takes into account t'Hooft interaction [8]:

$$f_0 = \sigma_u \sin\alpha + \sigma_s \cos\alpha, \tag{4}$$

$$\sigma = \sigma_u \cos\alpha - \sigma_s \sin\alpha. \tag{5}$$

Isovector meson a_0 consists of light quarks u and d only.

III. THE DECAYS $a_0 \to \omega\gamma$, $a_0 \to \rho\gamma$, $f_0 \to \omega\gamma$, $f_0 \to \rho\gamma$

In this paper we consider the decays of f_0 and a_0 mesons:

$$\begin{aligned} a_0(p) &\to \omega(q) + \gamma(k_1), \\ a_0(p) &\to \rho(q) + \gamma(k_1), \\ f_0(p) &\to \omega(q) + \gamma(k_1), \\ f_0(p) &\to \rho(q) + \gamma(k_1), \end{aligned}$$

$$p^2 = M_S^2, \qquad q^2 = M_V^2, \qquad k_1^2 = 0, \tag{6}$$

where $M_S = M_{f_0,a_0} = 980$ МэВ is the mass of decaying scalar meson [7] and $M_V = M_{\omega,\rho}$ is the mass of vector meson [7]. The matrix element in general case has the form:

$$M_i = e\,\frac{g_\rho}{2} A_i \left(q_\nu k_{1\mu} - g_{\mu\nu}\left(q k_1\right)\right) e_\gamma^\nu e_V^\mu, \tag{7}$$

$$e_\gamma = e\left(k_1\right), \qquad e_V = e\left(q\right), \tag{8}$$

where $i = \{a_0 \to \omega\gamma, a_0 \to \rho\gamma, f_0 \to \omega\gamma, f_0 \to \rho\gamma\}$, and the quantity A_i contains all coupling constants and the dynamical information of the process. Then the radiative decay width takes the form:

$$\Gamma_i = \frac{\alpha\left(M_S^2 - M_V^2\right)^3}{32 M_S^3} g_\rho^2 \left|A_i\right|^2. \tag{9}$$

In order to calculate the contributions of quark and meson loops to the coefficients A_i it is necessary to have information about the quark-meson and the meson-meson vertexes. The quark-meson vertexes are given in the Lagrangian (1). The meson-meson vertexes are calculated in a standard way of the local NJL model and expressed in terms of logarithmically divergent integrals which appears from the quark loops [5]. As a result the expression for this vertexes takes the form [1, 2]:

$$\begin{aligned} g_{\sigma_s K^+K^-} &= 2\sqrt{2} g_{\sigma_s}\left(2m_s - m_u\right) Z, \\ g_{\sigma_u K^+K^-} = g_{a_0 K^+K^-} &= 2\left(2m_u - m_s\right) g_{\sigma_u} Z, \\ g_{\sigma_u \pi^+\pi^-} &= 4 m_u g_{\sigma_u} Z, \\ g_{\omega^\mu K^+K^-} &= g_{\rho^\mu K^+K^-} = \frac{g_\rho}{2}\left(p_+ - p_-\right)^\mu, \\ g_{\rho^\mu \pi^+\pi^-} &= g_\rho \left(p_+ - p_-\right)^\mu. \end{aligned}$$

A. The decays $a_0 \to \omega\gamma$ and $a_0 \to \rho\gamma$

Let us consider the decay of isoscalar scalar meson a_0 into $\omega\gamma$. In this process the quark loop and the kaon loop give contribution. The contribution of pion loop is absent because the vertex $a_0 \to 2\pi$ is forbidden.

The quark loop contributions contain only u and d quarks. Then we obtain:

$$M^{(u,d)}_{a_0\to\omega\gamma} = e\,\frac{g_\rho}{2} C^{(u,d)}_{a_0\to\omega\gamma} \int \frac{d^4k}{i\pi^2} \frac{Sp\left[\left(\hat{q}+\hat{k}+m_u\right)\left(\hat{k}-\hat{k}_1+m_u\right)\hat{e}_\gamma\left(\hat{k}+m_u\right)\hat{e}_\omega\right]}{\left((q+k)^2-m_u^2\right)\left(k^2-m_u^2\right)\left((k-k_1)^2-m_u^2\right)}, \quad (10)$$

where $C^{(u,d)}_{a_0\to\omega\gamma} = 3 g_{\sigma_u}$, where 3 takes into account the color factor N_c and the sum of quark charge absolute values. We drop the imaginary part of quark loop which corresponds to condition of "naive"quark confinement. The confirmation of this prescription can be found in the paper [9]. Using standard Feynman method of denominator unification and following loop momentum integration we get:

$$M^{(u,d)}_{a_0\to\omega\gamma} = e\,\frac{g_\rho}{2} C^{(u,d)}_{a_0\to\omega\gamma} \mathrm{Re}\left(I_u\right)\left(q_\nu k_{1\mu} - g_{\mu\nu}\left(qk_1\right)\right) e^\nu_\gamma e^\mu_\omega, \quad (11)$$

where

$$I_u = 4m_u \int_0^1 dx \int_0^{1-x} dy \frac{1 + y(3x^2-x) - y^2 x(1-x)}{m_u^2 - y(1-y)(1-x)k_1^2 - y(1-y)xq^2 - x(1-x)y^2p^2 + i\epsilon}, \quad (12)$$

Now let us consider the contribution which comes from the kaon loop. Note that here we have two meson diagrams. One is the triangle loop diagram where vector meson and photon are in different points of meson loop:

$$M^{(K)}_{a_0\to\omega\gamma} = e\,\frac{g_\rho}{2} C^{(K)}_{a_0\to\omega\gamma} \int \frac{d^4k}{i\pi^2} \frac{(q+2k)_\mu (2k-k_1)_\nu e^\mu_\gamma e^\nu_\omega}{\left((q+k)^2-M_K^2\right)\left(k^2-M_K^2\right)\left((k-k_1)^2-M_K^2\right)}, \quad (13)$$

where $C^{(K)}_{a_0\to\omega\gamma} = g_{a_0K^+K^-}$. The second one is the diagram with two vertexes where the vector meson and photon are in the same vertex. This diagram contains only Lorenz-structure $g_{\mu\nu}$. As a result we obtain the gauge invariant form of meson amplitude similar to (8):

$$M^{(K)}_{a_0\to\omega\gamma} = e\,\frac{g_\rho}{2}C^{(K)}_{a_0\to\omega\gamma}I_K\left(q_\nu k_{1\mu} - g_{\mu\nu}\left(qk_1\right)\right)e^\nu_\gamma e^\mu_\omega, \tag{14}$$

where

$$I_K = \int\limits_0^1 dx \int\limits_0^{1-x} dy \frac{4y^2x(1-x)}{M_K^2 - y(1-y)(1-x)k_1^2 - y(1-y)xq^2 - x(1-x)y^2p^2 + i\epsilon} \tag{15}$$

Let us emphasize that gauge invariance leads to a finite expression for all this loop diagrams. More detail calculation of the similar diagrams can be found in [1].

Then the total amplitude of the process $a_0 \to \omega\gamma$ is equal

$$M_{a_0\to\omega\gamma} = e\,\frac{g_\rho}{2}A_{a_0\to\omega\gamma}\left(q_\nu k_{1\mu} - g_{\mu\nu}\left(qk_1\right)\right)e^\nu_\gamma e^\mu_\omega, \tag{16}$$

where

$$A_{a_0\to\omega\gamma} = 3g_{\sigma_u}\mathrm{Re}\left(I_u\right) + g_{a_0K^+K^-}I_K = -1.78374 + 0.159415 = -1.62433. \tag{17}$$

Then the decay width is equal:

$$\Gamma_{a_0\to\omega\gamma} = 114.7 \text{ кэВ}.$$

The decay $a_0 \to \rho\gamma$ can be considered in a complete analogy with the decay $a_0 \to \omega\gamma$. Let us note that here quark contribution to the amplitude is three times smaller than quark contribution to the amplitude of the decay with ω-mesons, while the kaon loops contributions are the same. As a result we get for the amplitude:

$$\begin{aligned} M_{a_0\to\rho\gamma} &= e\,\frac{g_\rho}{2}A_{a_0\to\rho\gamma}\left(q_\nu k_{1\mu} - g_{\mu\nu}\left(qk_1\right)\right)e^\nu_\gamma e^\mu_\rho, \\ A_{a_0\to\rho\gamma} &= g_{\sigma_u}\mathrm{Re}\left(I_u\right) + g_{a_0K^+K^-}I_K = -0.598209 + 0.156921. \end{aligned} \tag{18}$$

Then the total decay width is:

$$\Gamma_{a_0\to\rho\gamma} = 8.47 \text{ кэВ}.$$

Note that in the both processes the main contribution comes from the quark loops.

B. The decays $f_0 \to \omega\gamma$ and $f_0 \to \rho\gamma$

Since the scalar meson $f_0(980)$ consists of two components: the component which consists of light quarks u and d – σ_u and the component which consists of strange

quarks – σ_s, we will calculate the contributions of these two components to the total amplitude of decays $f_0 \to \omega(\rho)\gamma$ separately.

Let us start from the decay $f_0 \to \omega\gamma$. The component σ_u contains contribution of the quark and the kaon loops. As in previous section for quark loop contribution to the amplitude with the σ_u we get:

$$A^{u,d}_{\sigma_u \to \omega\gamma} = -0.59, \tag{19}$$

The contribution of pion loop here vanishes since the decay $\omega \to 2\pi$ is absent. The kaon loop contribution is

$$A^{K}_{\sigma_u \to \omega\gamma} = A^{K}_{a_0 \to \omega\gamma} = 0.0998425.$$

Concerning σ_s component only kaon loop contribution is present in the amplitude:

$$A^{K}_{\sigma_s \to \omega\gamma} = -1.08958, \tag{20}$$

As a result for total decay amplitude we obtain:

$$A_{f_0 \to \omega\gamma} = \sin\alpha(A^{u,d}_{\sigma_u \to \omega\gamma} + A^{K}_{\sigma_u \to \omega\gamma}) + \cos\alpha A^{K}_{\sigma_s \to \omega\gamma} = -0.102239 + (-1.06636) = -1.1686.$$

The total decay width is:

$$\Gamma_{f_0 \to \omega\gamma} = 59.67 \text{ кэВ}.$$

Let is now consider the decay $f_0 \to \rho\gamma$. The quark loop contribution to the σ_u component here is three times bigger that in the case of $f_0 \to \omega\gamma$ decay and is equal:

$$A^{(u,d)}_{\sigma_u \to \omega\gamma} = -1.80414, \tag{21}$$

The meson contribution to the σ_u component decay consists of kaon and pion loops:

$$\begin{aligned} A^{\pi}_{\sigma_u \to \rho\gamma} &= 0.22272 - 0.0276848i, \\ A^{K}_{\sigma_u \to \rho\gamma} &= 0.137775. \end{aligned}$$

The decay of σ_s component comes through the kaon loop only

$$A^{K}_{\sigma_s \to \rho\gamma} = -1.07395. \tag{22}$$

As a result for total amplitude of the decay $f_0 \to \rho\gamma$ we get:

$$\begin{aligned} A_{f_0 \to \rho\gamma} &= \sin\alpha \left(A^{(u,d)}_{\sigma_u \to \omega\gamma} + A^{\pi}_{\sigma_u \to \rho\gamma} + A^{K}_{\sigma_u \to \rho\gamma}\right) + \cos\alpha A^{K}_{\sigma_s \to \rho\gamma} = \\ &= \sin\alpha\,(-1.443 - 0.0276848i) + \cos\alpha \cdot (-1.07395) = \\ &= (-0.29632 + 0.00568i) + (-1.05106) = -1.34752 - 0.00568507i. \end{aligned}$$

The total decay width of the process $f_0 \to \rho\gamma$ is:

$$\Gamma_{f_0 \to \rho\gamma} = 81.435 \text{ кэВ}.$$

Let us note that in both decays $f_0(980)$ into $\rho\gamma$ and $\omega\gamma$ the main contribution to the amplitude comes from the kaon loop connected with the σ_s component of f_0 meson.

IV. CONCLUSION

Fulfilled calculation shown the important role of both the quark loop and meson loop for description of the radiative decays of the scalar mesons. It is interesting to emphasize that for the description of the radiative decays of isoscalar scalar meson $f_0(980)$ kaon loops play a dominant role. In paper [1] it was shown in the decay $\phi \to f_0(980)\gamma$ and in paper [2] on the basis of two-photon decay $f_0 \to 2\gamma$. Here we also see the dominant role of the kaon loop in the processes $f_0 \to \omega(\rho)\gamma$. This explains the success of other models, which are based on the assumption of scalar meson structure as a kaon molecule [4] or as a four-quark state [3]. However in the decays of other scalar mesons, for instance $a_0(980) \to 2\gamma$ [2] and $a_0 \to \omega(\rho)\gamma$, the quark loops play the dominant role. But meson loops also give a comparable contributions. Used here NJL model allows us to take into account correctly the contributions of both diagrams.

Note that the results obtained in the framework of NJL model in papers [1, 2] were in a satisfactory agreement with the experimental data. This allows us to hope that the predictions obtained here will also be in an adequate agreement with the future experiments.

In the future we hope to use the obtained here results for the description of the processes $e^+e^- \to SV$, where $V = \rho, \omega$ and $S = a_0, f_0$.

V. ACKNOWLEDGEMENTS

The authors would like to thank V. N. Pervushin for valuable discussions. E. A. K. and Yu. M. B. are grateful for support of INTAS (grant no. 05-1000008-8528).

[1] Y. M. Bystritskiy, M. K. Volkov, E. A. Kuraev, E. Bartos, and M. Secansky, Phys. Rev. **D77**, 054008 (2008), 0712.0304.
[2] Y. L. Kalinovsky and M. K. Volkov, (2008), 0809.1795.
[3] N. N. Achasov and V. N. Ivanchenko, Nucl. Phys. **B315**, 465 (1989).
[4] J. D. Weinstein and N. Isgur, Phys. Rev. **D41**, 2236 (1990).
[5] M. K. Volkov, Fiz. Elem. Chast. Atom. Yadra **17**, 433 (1986).
[6] M. K. Volkov and A. E. Radzhabov, Phys. Usp. **49**, 551 (2006).
[7] Particle Data Group, C. Amsler *et al.*, Phys. Lett. **B667**, 1 (2008).
[8] M. K. Volkov, M. Nagy, and V. L. Yudichev, Nuovo Cim. **A112**, 225 (1999), hep-ph/9804347.
[9] V. N. Pervushin, Riv. Nuovo Cim. **8N10**, 1 (1985).